How difficult it would be to detect Cosmic Neutrino Background?

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Based on the paper with Rimas Lazauskas and Cristina Volpe in J. Phys. G. 38, 025001 (2008).

Outline:

- 1) Number density of the Cosmic Neutrino Background
- 2) Clustering of CNB
- 3) Using coherence (or not)
- 4) Detection with radioactive targets

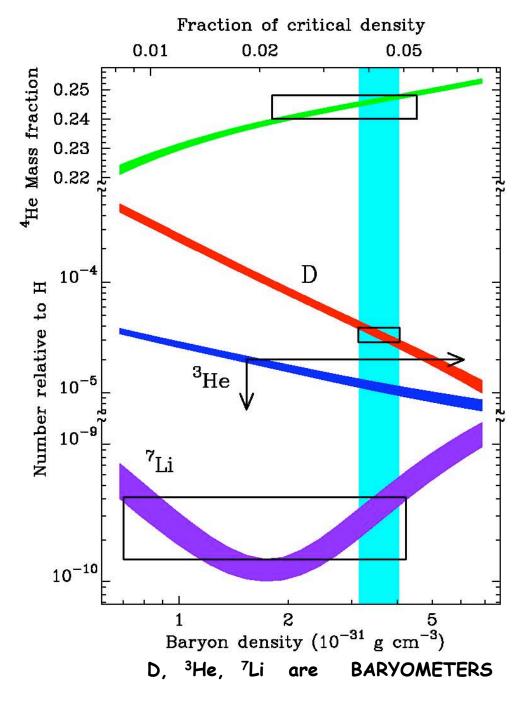
Hot Big-Bang Cosmology (concordance model of cosmology)

explains everything we know about the evolution of the Universe since early times with remarkable accuracy.

In particular, two totally independent ways of determining the baryon average density (or the ratio of baryons to photons), one from the **Big-Bang Nucleosynthesis** (first few minutes), and the other one from analysis of the temperature fluctuations of the **Cosmic Microwave Background** (~400 ky) agree very well.

Both sets of data also agree (albeit with large error bars) on the prediction that <u>relativistic neutrinos of ~3 flavors</u> were present at those epochs. Since these neutrinos have not interacted since that time with anything, they should be around us until now.

BBN - Predicted Primordial Abundances



BBN probes the Universe at ~20 minutes (time when deuteron density reaches its final value)

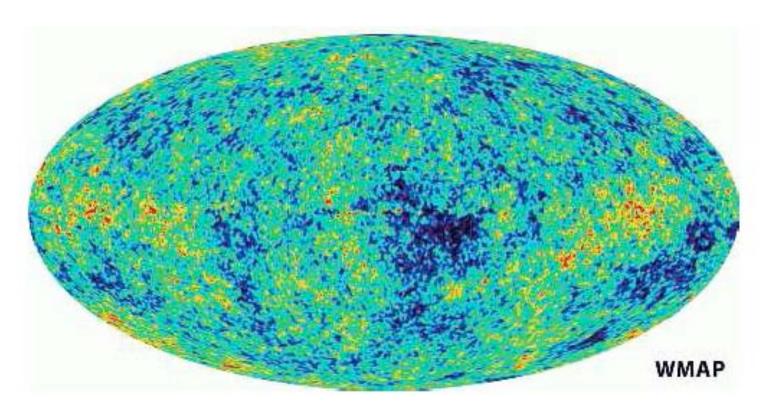
 $ho_{\rm B}{}^{\rm BBN} = 3.8 \pm 0.2 \times 10^{-31} g \ \text{cm}^{-3}$ (Freedman & Turner, 2003)

 $N_v = 2.4 \pm 0.4$ (from D, ⁴He) (Steigman 2008)

Note that $3.8 \times 10^{-31} \text{g/cm}^3$ is the same as $n_B = 2.2 \times 10^{-7}$ nucleons/cm³, which in turn is the same as $n_B/n_Y = 6 \times 10^{-10}$, the usual value.

CMB temperature fluctuations from WMAP

(snapshot at 380 k years)



Analysis gives ρ_B^{CMB} = 4.0±0.6 x 10⁻³¹ g cm⁻³ (Freedman & Turner, 2003)

 $N_v = 3.1_{-1.7}^{+2.2}$ (Steigman 2008, uses also LSS data)

In the radiation dominated epoch energy density and time evolve as

$$\rho = 3c^2/(32\pi G_N) t^{-2};$$
 kT = $[45 h^3 c^5/(32\pi^3 G_N g_s^*)]^{1/4} t^{-1/2},$ kT/MeV ~ $(t/s)^{-1/2}$

Where $g_s^* = 1 + 7/4 + 3x7/8$ (photons, electrons, 3 neutrino flavors)

Neutrinos decouple when the expansion rate exceeds the interaction rate:

$$\sigma \sim G_F^2 (kT)^2$$
, $n_v \sim (kT)^3$, $t_v = (n_v \sigma v)^{-1} \sim G_F^{-2} (kT)^{-5}$

$$t_{\text{expansion}} \sim G_N^{-1/2} (kT)^{-2}$$

 $(t_v - interval between weak interactions, t_{exp} - characteristic expansion time)$

From
$$t_v = t_{exp} \rightarrow kT \sim 1 \text{ MeV}$$
, $t_{decoupling} \sim 1 \text{ second}$

(detailed calculations give kT(v_e) ~ 2 MeV, kT(v_μ , v_τ) ~ 3 MeV),

While in equilibrium the number density of each Majorana neutrino flavor is proportional to the photon number density

 $n_v/n_v = 3/4$ (for relativistic Fermi and Bose gases)

At t ~ 10 s, e^+ and e^- annihilate increasing n_{v} .

That process conserves entropy, s ~ ρ/T

Thus the photon density n_y increases by the factor (1 + 2x7/8) = 11/4

 $n_v = (4/11)(3/4) n_v \sim 112$ neutrinos of each Majorana flavor /cm³

and $T_v/T_v = (4/11)^{1/3} = 0.71$; $T_v = 1.94 \text{ K} = 1.67 \times 10^{-4} \text{ eV}$

Reminder: Few textbook formulas re distribution functions of particle momenta

in thermal equilibrium

$$f_i^{eq}(p,T) = \left[\exp\left(rac{E_i - \mu_i}{T}
ight) \mp 1
ight]^{-1}$$

		relativistic Bose-Einstein	relativistic Fermi-Dirac	nonrelativistic
number density	n	$\frac{\zeta(3)}{\pi^2}gT^3$	$\frac{3}{4} \frac{\zeta(3)}{\pi^2} gT^3$	$g\left(\frac{mT}{2\pi}\right)^{3/2}e^{-m/T}$
energy density	ρ	$\frac{\pi^2}{30}gT^4$	$\frac{7}{8}\frac{\pi^2}{30}gT^4$	mn
pressure	p	$\frac{\rho}{3}$		$nT \ll \rho$
mean energy	$\langle E \rangle$	2,701 <i>T</i>	3,151T	$m + \frac{3}{2}T$
•				

$$n = g_i \int \frac{d^2 \vec{p}}{(2\pi)^3} f_i(p, T) \qquad \rho = g_i \int \frac{d^2 \vec{p}}{(2\pi)^3} E_i f_i(p, T)$$

$$p = g_i \int \frac{d^2 \vec{p}}{(2\pi)^3} \frac{p^2}{3E_i} f_i(p, T) \qquad \langle E \rangle = \rho/n$$

These are then <u>firm</u> predictions of the Hot Big-Bang Cosmology:

Neutrino number density = 112 neutrinos/cm³ for each flavor, i.e., **56 neutrinos and 56 antineutrinos of each flavor**

Neutrino temperature = $1.94 \text{ K} = 1.67 \times 10^{-4} \text{ eV}$

If one could confirm (or find deviations) from these predictions, one would test the theory at $t \sim 1$ sec, $T \sim 1$ MeV, much earlier and hotter than the tests based on BBN and CMB.

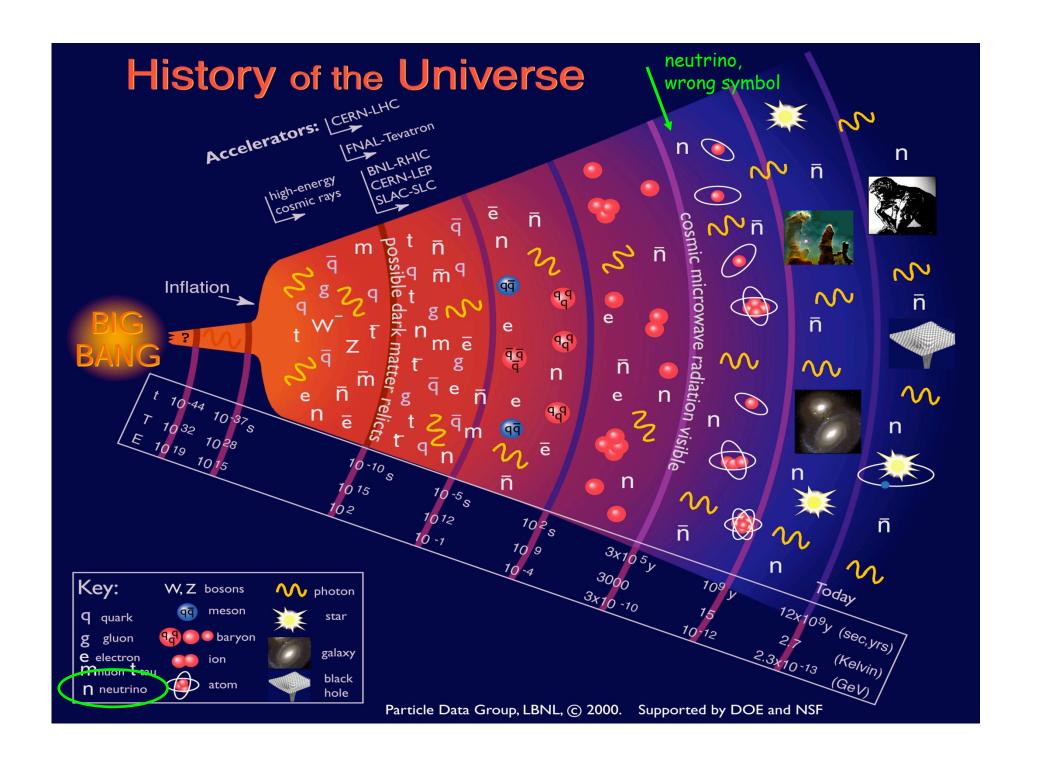
In order to motivate the need for CNB detection even more, lets compare the time, temperature, and redshift of different epochs:

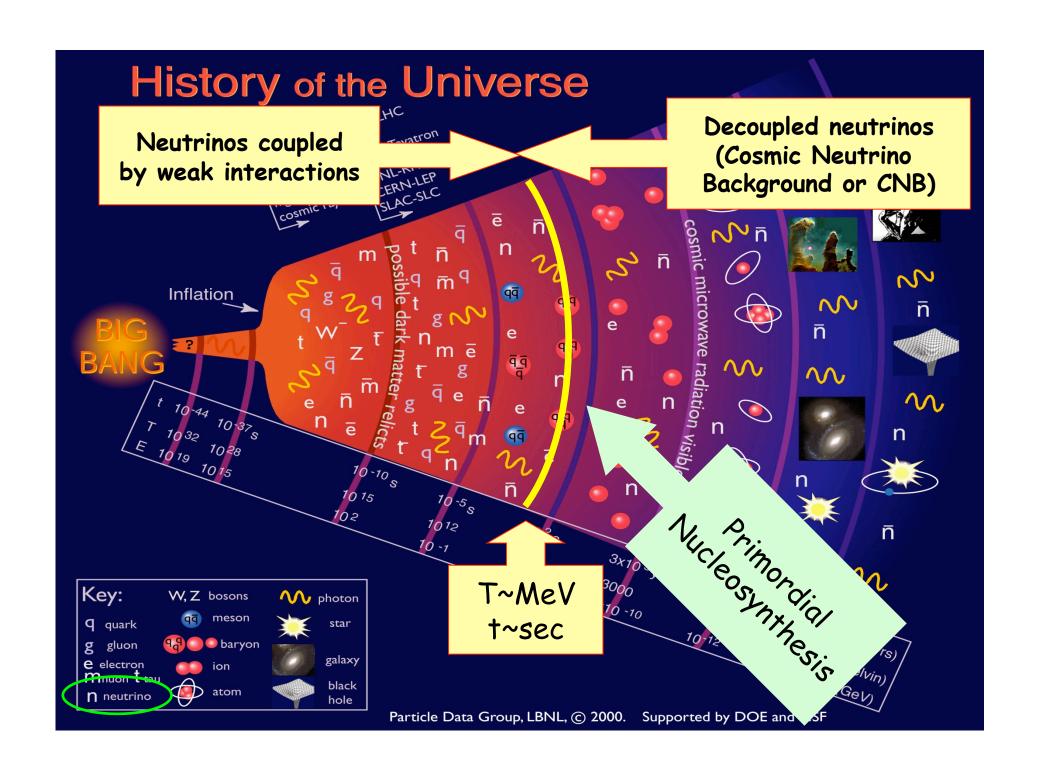
Epoch	time	Temperature	Z
CMB	3.8×10^{5} y	0.26 eV	1100
BBN	100-1000s	0.115-0.036MeV	$(4.9-1.8)\times10^8$
CNB	~0.18s	~2 MeV	~1.2×10 ¹⁰

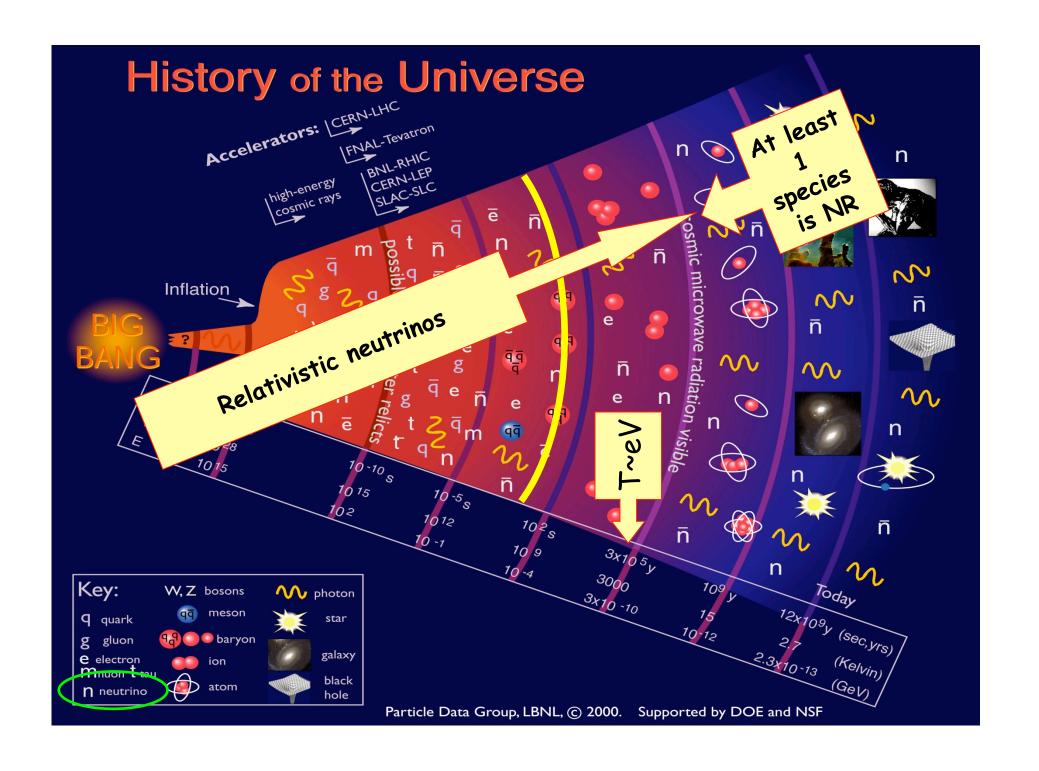
In other words, by observing CNB we would extend our observational capabilities by almost two orders of magnitude in in temperature and redshift and by almost four orders of magnitude by time since Big Bang. Note also that the furthest galaxy we see has $z\sim7$.

Thus evidence for CNB existence is so far indirect, based only on cosmological arguments and measurements, i.e., on the analysis of big bang nucleosynthesis (few minutes) and on the spectrum of CMB anisotropies combined with the large scale matter mower spectrum (400k years).

We would like to have more direct evidence, based on the weak interactions of CNB, and sensitive to the CNB in the present epoch and in our local neighborhood.







Clustering neutrino density enhancement

Massive particles become nonrelativistic when their mass exceeds the temperature of the Universe. From than on they can become bound, i.e., concentrate in structures of various sizes. Their densities in these structures can far exceed the average density derived from cosmological measurements and arguments.

The overall energy density (critical density for Ω = 1) of the Universe is ρ_c = 1.05x10⁴ h_{100}^2 eV/cm³ ~ 5 keV/cm³ (since h_{100} ~ 0.73)

component	average ρ (keV/cm ³)	Structure	Enhancement
baryons	0.2	galaxy(disk)	~5×10 ⁶
dark matter	1.0	galaxy(halo)	~3×10 ⁵
Neutrinos	112($\Sigma m_{\nu}/\text{keV}$)	clusters	~100

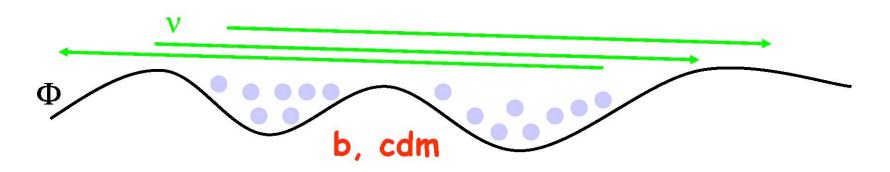
I assumed that neutrinos will concentrate in clusters of ~ 5 Mpc size with the total mass of ~ 10^{15} M $_{\odot}$ and that their enhancement in them will be similar to the average enhancement of baryons and cold dark matter.

Note that $\Omega_{\rm v}/\Omega_{\rm baryon} \sim 112$ (m_v/eV) / 200 eV ~ 0.5 (m_v/eV) for each flavor. I assumed that this ratio remains fixed in the structures where both neutrinos and baryons cluster.

Note also, that the energy density, and naturally also the number density of neutrinos scales as R^{-3} , where R is the characteristic size of of the clustering region

Neutrinos are natural Hot Dark Matter (HDM)candidates

Neutrino Free Streaming



An alternative estimate of the enhancement $n_v/\langle n_v \rangle$ is obtained by considering the HDM clustering with a velocity dispersion v (Peebles):

$$n_v/\langle n_v \rangle \approx v^3 m_v^3/(2\pi)^{3/2} = 330 (v/500 \text{ km/s})^3 (m_v/eV)^3$$

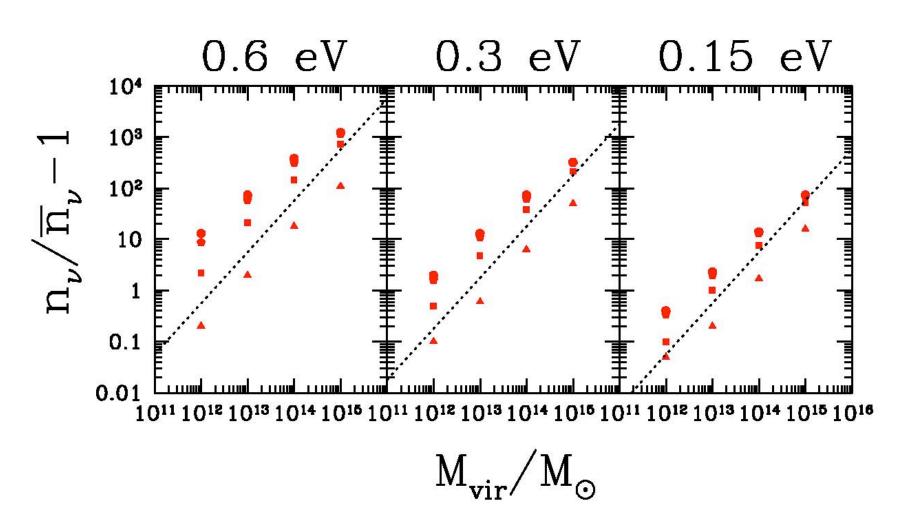
Obtained for $\langle n_y \rangle = 110$ cm⁻³ neutrino average number density.

Thus this estimate agrees with our previous $n_v/\langle n_v \rangle \approx 100$ (as far as the order of magnitude is concerned)

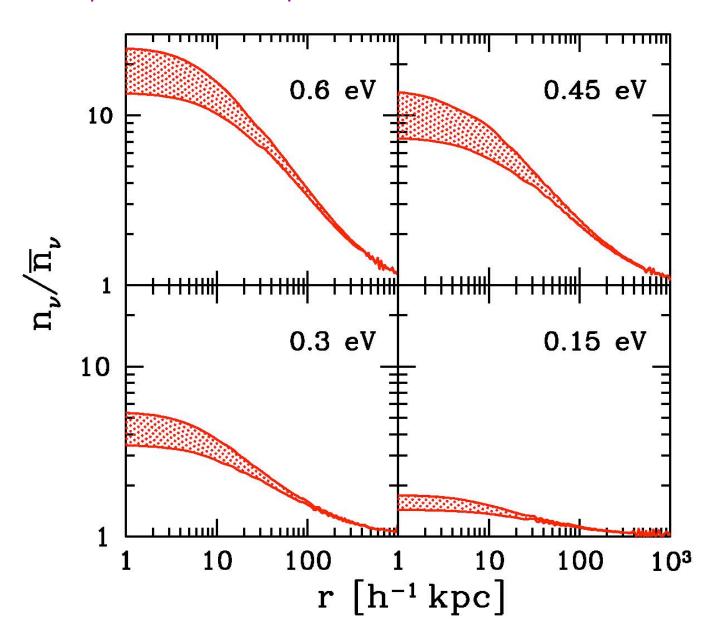
Dependence of the overdensity on the mass of the cluster and on the neutrino mass (from Ringwald & Wong, 04)

The red symbols indicate different distances from the cluster center, \triangle are for r = 1 Mpc/h.

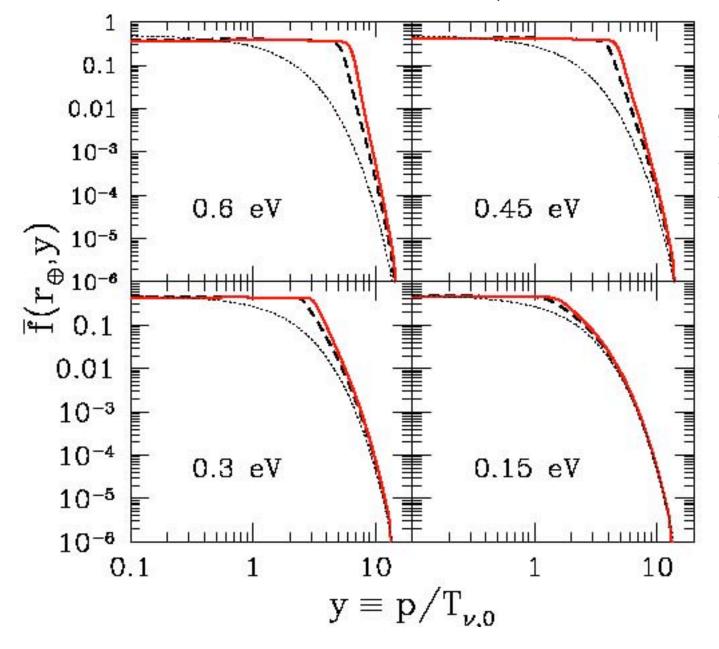
For $M_{vir} = 10^{15} M_{\odot}$, $m_v > 0.3$ eV our estimate $n_v / \langle n_v \rangle = 100$ looks OK



Clustering evaluation for the Milky Way (Ringwald & Wong 04) At 8 kpc the overdensity is less than what we estimated.

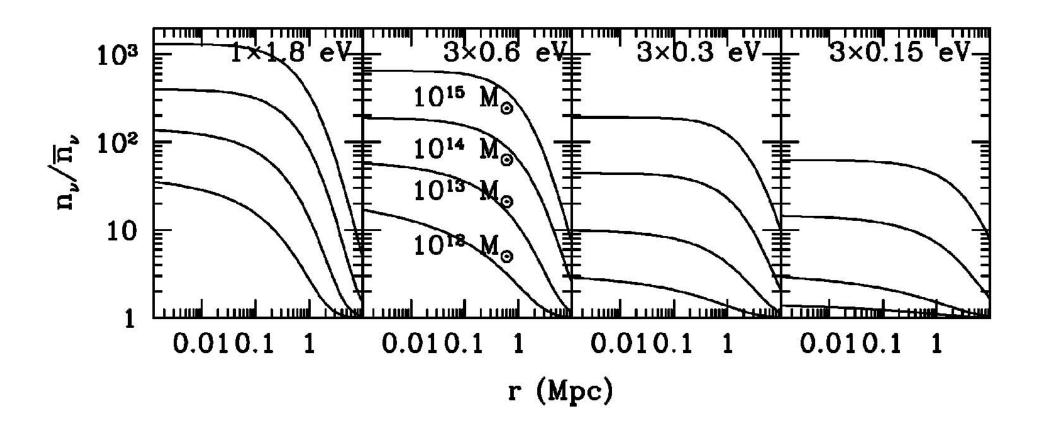


Local and present calculated CNB momentum distribution (Ringwald & Wong, 04) Full and dashed, two assumed distributions in Milky Way, dotted, relativistic Fermi-Dirac distribution. Note the relatively small deviations from F-D.



Neutrino momenta are almost isotropic; the Earth is moving through the CNB sea with v ~ 10⁻³c Another calculated neutrino clustering magnitudes (Singh & Ma, Phys. Rev. **D67**, 023506 (2003))

Shown is the dependence on the cluster size and neutrino mass. Here, for heavier neutrinos and larger clusters substantial number density enhancement occurs.



How do we detect Cosmic Neutrino Background (CNB)?

The first idea, from ~1980 when people believed that m_v ~ 30 eV, was to use the coherent scattering on macroscopic objects.

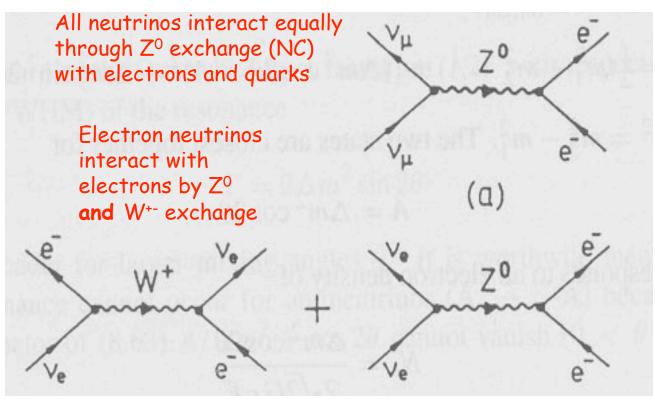
de Broglie wavelength $\lambda_v = h/p_v \sim 2.4 \text{ mm}$ (for $p_v \sim 3T_v$)

A sphere with d = λ_{ν} contains ~ 10^{21} nucleons. If neutrinos interact coherently with all of them, it should help a lot.

To describe the reflection or refraction on a thin foil, use the concept of index of refraction

$$n = 1 + N \lambda_v^2 f(0)/2\pi$$
,

where N is the number of density of target atoms and f(0) is the forward scattering amplitude. In order to evaluate n-1, the deviation of index of refraction from unity, proceed exactly the same way as in the treatment of the MSW effect for matter neutrino oscillations, namely evaluate these graphs:



Thus
$$n-1 = \pm [G_F \ N \ (3Z - A)]/(2^{3/2} \ T_v)$$
 for $v_e \ (\overline{v}_e)$

n-1 = \pm [G_F N (Z - A))]/(2^{3/2} T_v) for v_{μ} , v_{τ} (\overline{v}_{μ} , \overline{v}_{τ}) where T_v is the kinetic energy of nonrelativistic neutrinos

For v_{μ} on gold $1-n \approx 10^{-7}$ (eV/m_{ν}) for $v_{\nu} = 500$ km/s and the critical scattering angle $\theta_c = [2(1-n)]^{1/2} \approx 1.5$ arcmin

Consider neutrinos with flux density j (neutrinos/sr cm² sec). Collision rate for area of 1 cm² with angles less than θ_c is $2\pi j$ θ_c and the momentum transfer is p_v θ_c

The pressure of the `neutrino wind' is then

$$dp/dt = 4\pi \rho_v N G_F (A-Z) /2^{1/2}$$

linear in G_F and independent of v_v (Opher, 74,82; Lewis, 80)

Unfortunately, this derivation is wrong !!!

Arguments against the effects linear in G_F :

(Cabibbo & Maiani, 82; Langacker, Leveille & Sheiman, 83)

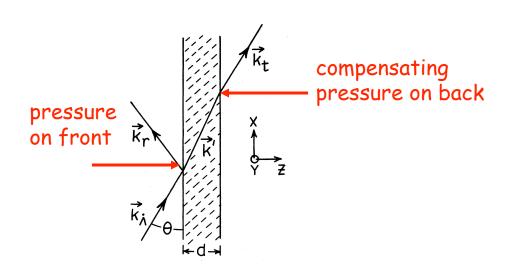
- 1) Another factor θ_c , projection of the area orthogonal to the neutrino ray is missing from dp/dt, thus dp/dt ~ $G_F^{3/2}$.
- 2) More importantly, the scattered wave penetrates into the foil and decays exponentially with $z_0 \approx \lambda/2\pi \theta_c$.

Thus, the pressure dp/dt vanishes if the foil thickness d is $d \leftrightarrow \lambda/\theta_c$

However, in our above example,

$$\lambda/\theta_c \approx 8 \text{ meters } x (m_v/eV)^{1/2}$$

Thus the pressure on the opposite surfaces of the foil cancel. The only effect left is ${}^{\sim}G_{F}^{2}$.



Another proposal to use coherence, this time $\sim G_F^2$ (Shvartsman, Braginski, Gershtein, Zeldovich, and Khlopov, 82)

Scatter relic neutrinos on spheres with $\mathbf{r}=\lambda$; use the virial motion of Earth with respect to the relic neutrinos, $\mathbf{v}\sim 300 \mathrm{km/s}$ and measure the force on such spheres.

Cross section $\sigma = G_F^2 m_v^2 k_L^2/\pi$, $k_L = 3Z-A$ (for v_e), A-Z (for v_μ, v_τ)

Force $F = 2n_v v m_v v \sigma N_A$ $(n_v = density of relic neutrinos, N_A = number of target atoms in each sphere)$

Acceleration of each sphere $a = F/m_{sphere}$ is independent of m_v .

Take iron spheres, assume clustering $n_v/\langle n_v \rangle = 100$,

a ~ 3×10^{-26} cm s⁻², F ~ 3×10^{-30} dyne This is ~13 orders of magnitude from the sensitivity of the current Dicke - Eotvos type experiments. Even though proposals for a substantial improvement of the sensitivity to small accelerations exist, they were never demonstrated.

Moreover, for Majorana neutrinos there is a further suppression of the acceleration by $(v/c)^2 \sim 10^{-6}$ for unpolarized targets, $(v/c) \sim 10^{-3}$ for polarized targets (see Hagmann, astro-ph/9902102)

Since none of these proposals work, by a huge margin, lets consider the usual way of detecting neutrinos, by charged current weak interactions.

The problems to solve:

- 1) Can one find an appropriate target?
- 2) How many target atoms can one use in practice?
- 3) What is the cross section, and is the event rate sufficient?
- 4) Can one separate the signal from background?

Each of these items is challenging, but it turns out that the needed technological improvements are *only(??!!)* one or few orders of magnitude each, so it is worthwhile to consider them in more detail.

Consider first the fluxes and corresponding (kinetic) energies (for each neutrino flavor):

Average With clustering (v=500kms⁻¹) Flux (cm⁻² s⁻¹)
$$0.8 \times 10^9$$
 x (eV/m_v) 2.8×10^{11} Kin. energy(eV) 1.2×10^{-7} x (eV/m_v) 1.4×10^{-6} (m_v/eV)

These fluxes can be compared to the solar pp neutrino flux of $\sim 6 \times 10^{10}/\text{cm}^2$ s, distributed over 420 keV, or to the v_e flux at a distance of 1 km from a power reactor, $4 \times 10^9/\text{cm}^2$ s spread over several MeV.

So, at the very small, sub eV, energies the CNB flux dominates over any other neutrino fluxes by a very large factor.

Since the momentum of the CNB $p_v \rightarrow 0$, we must consider only exothermic reaction, i.e., reactions on unstable targets. What is the behavior of the cross section when $p_v \rightarrow 0$?

The well known endothermic reaction has threshold (recoil neglected) E_{\rm thr} = \bar{\nu}_e + p \to e^+ + n and cross section

$$\frac{d\sigma}{d\cos\theta}=\bar{G}^2E_ep_e[(f^2+3g^2)+(f^2-g^2)v_ev_\nu\cos\theta]$$
 The positron energy with $\bar{G}=G_F\cos\theta_C/\sqrt{2\pi}$.

What about the exothermic (hypothetical, there are no free neutrons) reaction $\nu_e + n \rightarrow e^- + p$ with $E_e = M_n - M_p + E_v$ which remains positive and $E_e \ge m_e$ even when $E_v \rightarrow 0$?

The cross section now contains 1/v,, which means that the rate, respectively. $\frac{d\sigma}{d\cos\theta} = \frac{\bar{G}^2}{v_\nu} E_e p_e [(f^2+3g^2)+(f^2-g^2)v_e v_\nu\cos\theta]$

(see Weinberg 62, Cocco, Mangano, Messina 07)

Naturally, the $1/v_v$ factor should be there even for the endothermic reactions, but becomes irrelevant since in that case $v_v \rightarrow c$ (=1 here). This is a general result for reactions with nonrelativistic projectiles (known long time ago for the case of slow neutrons).

Perhaps the factor $1/v_v$ deserves a more detailed explanation:

Standard expression for the cross section on free nucleons at low neutrino energies is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^2} = \frac{G_F^2 \cos^2 \theta_C}{\pi} \frac{|\mathcal{M}|^2}{\left(s - M_p^2\right)^2}$$
 Where
$$|\mathcal{M}|^2 = M_n M_p E_\nu E_e [(f^2 + 3g^2) + (f^2 - g^2) v_e v_\nu \cos \theta],$$
 and
$$(s - M_p^2)^2 \to [s - (M_p + m_\nu)^2] [s - (M_p - m_\nu)^2] = 4 M_p^2 p_\nu^2,$$
 while
$$\frac{\mathrm{d}q^2}{\mathrm{d}\cos \theta} = 2 p_\nu p_e.$$

And now putting everything together one gets, since $v_v = p_v/E_v$

$$\frac{d\sigma}{d\cos\theta} = \frac{\bar{G}^2}{v_{\nu}} E_e p_e [(f^2 + 3g^2) + (f^2 - g^2)v_e v_{\nu} \cos\theta]$$

Consider now reactions on unstable nuclear targets A_7

$$v_e + A_Z \rightarrow e^- + A_{Z+1}$$
 or $v_e + A_Z \rightarrow e^+ + A_{Z-1}$

where the allowed β^{\pm} decay of $A_{Z\pm 1}$ is characterized by the known nuclear matrix element $|\mathbf{M}_{\text{nucl}}|^2 \approx 6300/\text{ft}_{1/2}$.

The cross section in cm² for these exothermic reactions is

$$\sigma = \sigma_0 \times \langle \frac{c}{v_\nu} E_e p_e F(Z, E_e) \rangle \frac{2I' + 1}{2I + 1}$$

$$\sigma = \sigma_0 \times \langle \frac{c}{v_{\nu}} E_e p_e F(Z, E_e) \rangle \frac{2I' + 1}{2I + 1}$$
 with
$$\sigma_0 = \frac{G_F^2 \cos^2 \theta_C m_e^2}{\pi} |M_{nucl}|^2 = \frac{2.64 \times 10^{-41}}{ft_{1/2}}$$

When $v_y \rightarrow 0$ the e[±] energies are monoenergetic $E_e = Q + m_e + m_y$

We can consider now the answer to our first question:

Can one find an appropriate target?

Clearly the unstable A_Z target should have halflife $t_{1/2}$ longer than the duration of the measurement, i.e., $t_{1/2} \ge years$.

It could be manmade, or it could exist in nature. However, natural radioactivity has $t_{1/2} \ge 10^9$ years.

The target A_Z should also have minimal possible $ft_{1/2}$ so that the cross section is as large as possible. This means that the superallowed decays, with $ft_{1/2} \sim 1000$ are preferred.

Now, lets consider the second question:

How many target atoms can one use in practice?

When reviewing possible targets, the tritium (3 H) clearly comes to mind. Its halflife $t_{1/2}$ = 12.3 years is just right, and $ft_{1/2}$ = 1143 is almost as small as the $ft_{1/2}$ for the free neutron decay.

The technology of production is well developed, and using as much as 1 Mcu (2.1x10²⁵ tritium atoms) is very challenging but appears to be technologically possible.

This corresponds to just \sim 100 g of pure tritium.

(Note, however, that the Karlsruhe facility, handling all tritium for the KATRIN experiment, as well as for ITER, is licensed for maximum only 20 g of tritium. KATRIN experiment will run with ~ 3 curies T_2 source, constantly recirculated.)

Alternative target, considered in the literature (Messina) is ¹⁸⁷Re.

It has Q_{β} = 2.4 keV, $T_{1/2}$ = 4.4 x 10¹⁰ years, ft_{1/2} = 1.6 x 10¹¹ ,

Hence one would need ~108 times more target material.

Small ¹⁸⁷Re detectors with good energy resolution exist (~mg).

If they can be made bigger, say few g at least, and if one could combine many of them, this might be an alternative CNB detection possibility.

Now the third question:

What is the cross section, and the event rate?

To estimate the relic neutrino velocity, lets neglect the virial motion and use $v_y/c \sim 3T_y/m_y$, with $T_y = 1.9$ K.

With this assumption $\sigma = 1.5 \times 10^{-41} \, (m_v/eV) \, cm^2$

The CNB capture rate per tritium atom is independent of m_{v} ,

R =
$$\sigma \times v_v \times n_v \approx 1.8 \times 10^{-32} \times n_v / \langle n_v \rangle \text{ s}^{-1}$$
 (independent of v_v)

And the number of events is

 $N_{v \, capt} \approx 830 \, yr^{-1} \, Mcu^{-1} \, for \, n_v / \langle n_v \rangle = 100$

So, the number of events would be reasonably large.

(this estimate is somewhat larger than in our paper;

I did not use round-off.)

Can we understand that it is possible to have a considerably larger neutrino capture rate with only ~100g of tritium compared with ~500 ton (fiducial) of scintillator in KamLAND?

Here are the ratios tritium/KamLAND:

Cross section ~100

Number of targets $\sim 5 \times 10^{-7}$

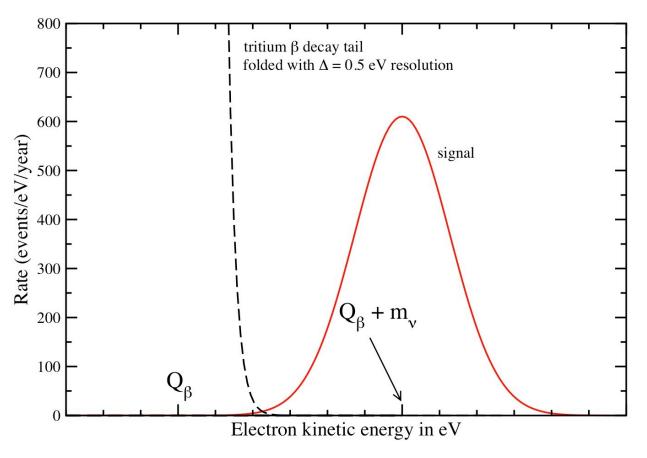
Flux ~10⁵

Total ~5

Finally, the last and most difficult question:

Can one separate the signal from background?

There are 3.7x10¹⁶ tritium β decays/s , and hence emitted electrons distributed over the energy interval 0 — Q_β - m_ν and smeared by the detector energy resolution. The fraction of electrons in the energy interval of width Δ just below the endpoint is ~ $(\Delta/Q_\beta)^3$



This is for $\Delta = 0.5$ eV $m_v = 1$ eV and $n_v / \langle n_v \rangle = 50$.

KATRIN-type spectrometer cannot be made any bigger



There are, thus, two challenging problems:

- 1) Can one filter out up to the $\sim 10^{16}$ electrons/s that have energies below the endpoint?
 - In KATRIN design the ratio between electrons in the window of planned 0.2 eV sensitivity and the total decay rate is $\sim 10^{15}$. So, the filter used in KATRIN will be almost capable to reach the required rejection ratio.
- 2) Can one reach the required energy resolution? And how the signal to background ratio depends on the resolution Δ and on the neutrino mass m_v ?

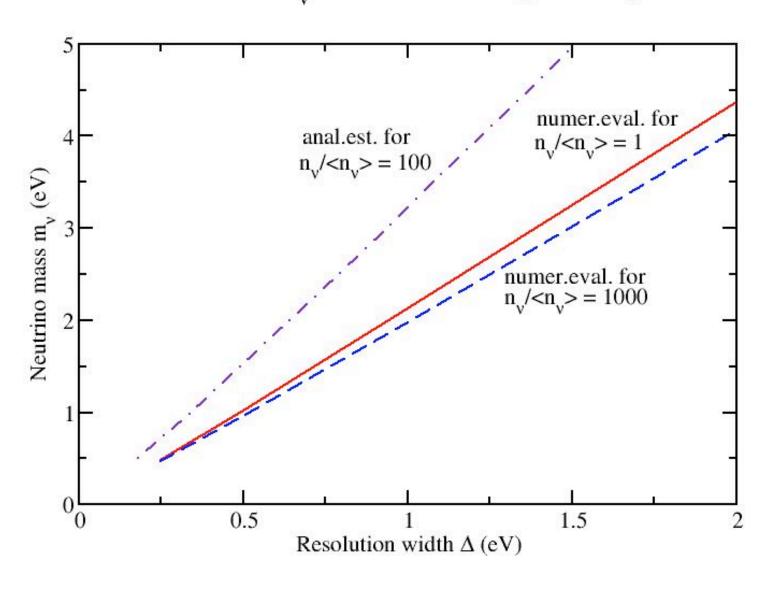
It turns out one can make an analytic estimate of the ratio

$$\lambda_{v}/\lambda_{\beta} = 6\pi^{2} n_{v}/\Delta^{3} \times (2\pi)^{1/2} e^{2z}$$
, $z = (m_{v}/\Delta)^{2}$

valid reasonably well as long as $m_v > \Delta$ (Cocco *et al.*). Note that this ratio is independent on the Q-value and on the β decay nuclear matrix element (hence also on $t_{1/2}$)

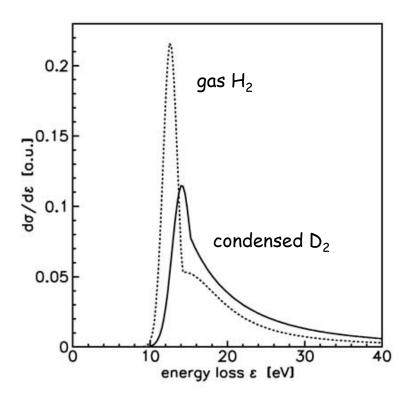
The analytic formula suggest that $m_v/\Delta \sim 3$ is needed, numerical evaluation gives $m_v/\Delta \sim 2$, a somewhat more favorable ratio.

Relation between m_v and Δ for which signal/background = 1



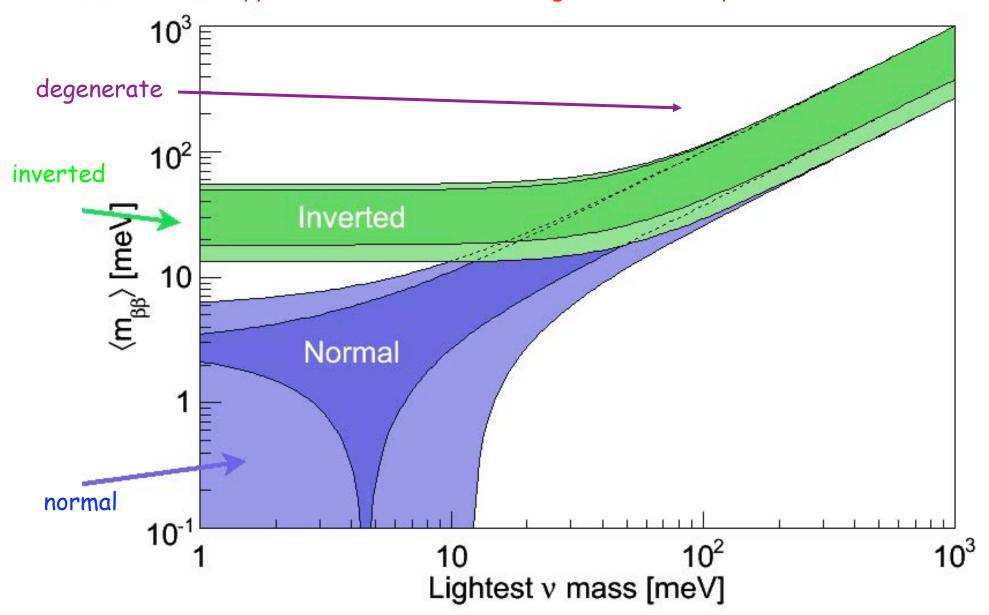
Here are potential killer problems:

- 1) Past and planned experiments use molecular T_2 . The rotational-vibrational states in the final ³HeT molecule are spread over ~0.5 eV. That essentially limits the achievable resolution. However, using atomic T would be very difficult.
- 3) Electrons scatter on T_2 with $\sigma=3x10^{-18}cm^2$. This limits the source column density and makes sources of 1kCu or more



impossible. New arrangement would be needed for stronger sources (see the idea in Monreal & Formaggio, arXiv:0904.2860).

Representation of the three different possible neutrino mass patterns. The method of detecting CNB discussed here appears to be very challenging, but with effort applicable for the case of degenerate mass pattern



Summary

- 1) We have discussed the challenges of detecting the primordial neutrinos (in particular the ν_e component) using the neutrino capture on radioactive nuclei, with emphasis on tritium as target.
- 2) Among the various technological challenges of such program, the requirement that the detector resolution is better that the neutrino mass by a factor 2 3, while at the same time dealing with extreme strong source strengths, appears to be the most difficult one to achieve. It essentially restricts the applicability of the discussed approach.
- 3) In the next few years a variety of approaches (KATRIN, cosmology & astrophysics, $0v\beta\beta$ decay) promise to reach sensitivity to $m_v \sim 0.2$ eV or even better. If one or all of these approaches find positive evidence, e.g.. if we can conclude that $m_v \geq 0.2$ eV, it would be certainly worthwhile, and perhaps even imperative, to pursue the indicated program vigorously.

Spares